The Universal Mechanics

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On the basis of Berkeley−Mach−Poincaré principle of relativity and in the assumption of absence of any specific (inertial) reference systems in empty space the new mechanics of bodies motion is constructed. This motion is determined only by the relative distances and velocities of bodies. It is shown that the mutual rotation of pair of particles related to almost immobile "distant stars" (galaxies) leads to centrifugal force origin.

Keywords: Gravitation; relativity; causality

1. Introduction

The concept of absolute space (aether) has been introduced by Newton at build-up of a mechanics[1]. This concept was undergone to criticism from outside Newton's many contemporaries, in particular Berkeley[2] who came to a conclusion about impossibility of existence of absolute space (aether) with its inertial reference systems, and also to understanding of that inertia should be caused by motion of bodies related to "distant stars". To a similar deduction later have come Mach[3] and Poincaré[4] who have specified in necessity of creation of a *relativistic mechanics*, i.e. a mechanics of the relative motion of bodies.

The given problem has not been solved neither in special, nor in common theories of relativity which still operated with concepts of absolute space. Numerous attempts to construct the theory in which inertia of rotation would be caused by influence of remote galaxies ("far stars"), have appeared unsatisfactory[5]. Thus, the problem of Berkeley−Mach−Poincaré is not solved in existing physical theories.

The theory named as a *universal mechanics* which initial principles essentially differ from usually used is stated below. It in particular concerns space-time properties of bodies motion.

At first, only relative motions of bodies are considered. Thus it is supposed that the space does not have any preferential (inertial) reference systems. Thereby the space is not considered as something absolute (aether) possessing to fix bodies motion.

Secondly, the causality principle is used for description of interaction of bodies, i.e. it is supposed that action of one body on another, located on distance r, happens in time r/c, where c – velocity of propagation of interactions (velocity of light). Thus velocity of propagation of interaction is constant and does not depend on character of movements of bodies that also means a space relativity. Causal character of interaction leads to irreversibility of movements of bodies in time. There is also no equality of actions of bodies against each other as bodies act on each other in different moments of time and, accordingly, at different distances.

Use of a principle of least action has allowed to discover a Lagrangian for a pair of particles and, further, for system of interacting particles.

In a limit of small velocities of bodies in comparison with c it is shown that a mutual rotation of two particles related to massive, remote, slowly moving bodies (galaxies) leads to origin of a centrifugal force. Also the plane motion of pair of particles with conservations of the mechanical momentum and the velocity of a inertia centre is realized – i.e. we obtain the equations of Newtonian mechanics. These result is a consequence of assumption of an isotropy of space allocation of large remote bodies. Therewith three various requirements of an isotropy are required.

2. Main principles

Space relativity. A causality principle

The space relativity means, in particular, that one particle in empty space cannot be characterized by a motion, and the only pair of particles cannot rotate relatively each other because of the impossibility of relative rotation in the absence of absolute space and other bodies.

Further it is supposed that the space does not have any specific systems of reference and directions, and also that action of any particle 2 on a particle 1 depends only on the relative distance r_{21} and the relative velocity v_{21} of this particles.

The causality principle means that if the reference system is associated with particle 1 and the time t_1, corresponding time t_2 when the particle 2 acts ("sends a signal") on a particle 1, is equation

$$t_2 = t_1 - \frac{r_{21}(t_1)}{c}. \tag{2.1}$$

Thus the space relativity also means that the velocity of interaction c does not depend on movements of particles.

In the reference system associated with particle 1 it is possible to define the velocity of a particle 2 related to particle 1:

$$v_{21}(t_1) = -\frac{dr_{21}(t_1)}{dt_1}. \tag{2.2}$$

Let's consider that action of a particle 2 on a particle 1 is characterized by a definite direction so, for example, at action of two particles 2 and 3 on a particle 1 it is possible to determine an angle $\theta_{1,23}$ between directions of action for simultaneously coming signals to a particle 1 in the moment t_1 (Fig. 2.1).

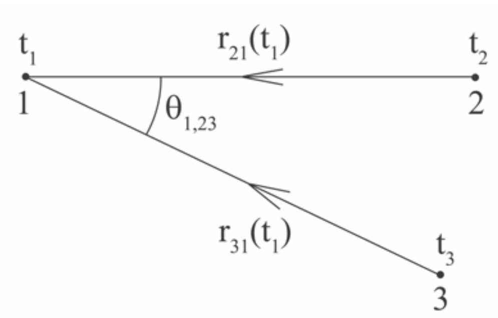

Fig. 2.1. Scheme of action of particles 2 and 3 on particle 1. Arrows show the direction of action.

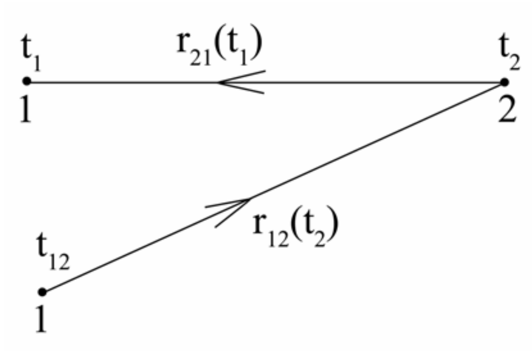

Fig. 2.2

The causality principle leads to difference between the times of action of a particle 2 on a particle 1 and contrary particle 2 on particle 1 (Fig. 2.2), so their actions against each other will appear, generally speaking, various that leads to an inequality of action and counteraction, and also to irreversibility of motion in time. We will note also that further in drawings it is necessary to view only the angles having physical sense. For example, on Fig. 2.2 "angle between directions" r_{12} and r_{21} has no physical sense, unlike Fig. 2.1 where the angle $\theta_{1,23}$ has physical sense.

Constancy of propagation velocity of a gravitational interaction (velocity of light)

This principle used in a special theory of relativity, in this case means that in the reference system of the particle $1'$ coinciding with a particle 1 and moving with velocity *V* in a direction from a particle 2 to particle 1, light propagation has the same velocity *c*, as in the system of a particle 1. Transformation of coordinates from system $1'$ to system 1can be written in the form of two-dimensional Lorentz–Poincaré transformation

$$dr'_{21} = \kappa\left(dr_{21} + V dt_1\right), \quad dt'_1 = \kappa\left(dt_1 + \frac{V}{c^2} dr_{21}\right), \quad \kappa = \left(1 - \frac{V^2}{c^2}\right)^{-1/2}, \qquad (2.3)$$

This transformations can be derived using the symmetry of transitions $1' \leftrightarrow 1$ and $V \to -V$, restriction of velocities of particles by quantity *c*, and also group properties of transformations at transition in a particle reference system $1'$, and further in a reference system of particle $1''$, moving with velocity rather V' related to particle $1'$ [6]. It is possible to present transition (2.3) in the form of unitary transformation:

$$\left(dr^i_{21}\right)' = P^{ij}\, dr^j_{21}, \quad P^{ij} = \kappa \begin{bmatrix} 1 & -i\frac{V}{c} \\ i\frac{V}{c} & 1 \end{bmatrix}. \qquad (2.4)$$

There is a summation on doubled index, and a two-component vector looks as

$$dr_{21}^{i} = \left(dr_{21};\, icdt_1\right). \tag{2.5}$$

Transformation (2.4) leaves invariable an interval

$$ds_{21} = \sqrt{-dr_{21}^{i}\, dr_{21}^{i}} = \sqrt{c^2 - v_{21}^2}\; dt_1\,. \tag{2.6}$$

Two-component vector of velocity we will note in a view:

$$u_{21}^{i} = -\frac{dr_{21}^{i}}{ds_{21}} = \left(1 - \frac{v_{21}^2}{c^2}\right)^{-1/2} \left(\frac{v_{21}}{c};\, -i\right). \tag{2.7}$$

The two-component vector of differentials of coordinates (2.5) suggests about existence of an integral vector

$$r_{21}^{i} = \left(r_{21};\, ic\left(t_1 - t_2\right)\right).$$

Substituting a relation (2.1) for t_2 in this expression we obtain:

$$r_{21}^{i} = \left(r_{21};\, i\, r_{21}\right). \tag{2.8}$$

From vectors (2.7), (2.8) it is possible to form the following invariants concerning transformation (2.4):

$$r_{21}^{i}\, r_{21}^{i} = 0,\; u_{21}^{i}\, u_{21}^{i} = -1,\; r_{21}^{i}\, u_{21}^{i} = r_{21}\sqrt{\left(1 + \frac{v_{21}}{c}\right) \Big/ \left(1 - \frac{v_{21}}{c}\right)}. \tag{2.9}$$

Formulas (2.3), (2.2) lead to following transformation of the relative velocity:

$$v'_{21} = \frac{v_{21} - V}{1 - \dfrac{v_{21} V}{c^2}}\;, \tag{2.10}$$

coinciding by the form with a velocity transformation rule in a special theory of relativity. However distance (2.8) transformation essentially differs from transformations of "lengths of rulers” in a special theory of relativity:

$$r'_{21} = r_{21}\sqrt{\left(1 + \frac{V}{c}\right) \Big/ \left(1 - \frac{V}{c}\right)}\,. \tag{2.11}$$

This expression is symmetrical concerning replacement $r_{21} \to r'_{21},\; V \to -V.$

Lagrangian and action function

The Lagrangian L_{21} characterizes action of a particle 2 on a particle 1, determining action function

$$S_{21} = \int L_{21}\left(v_{21},\, r_{21}\right) dt_1\,, \tag{2.11}$$

which should have minimum on real trajectories and is invariant concerning transformation (2.4). We will be interested only in a gravitational interaction when there are no other fields. Thus action function can depend only on invariants (2.6), (2.9). Then the invariance requirement of S_{21} leads to a following view of a Lagrangian:

$$L_{21}(v_{21},\ r_{21}) = \psi\left(r_{21}^{i}\, u_{21}^{i}\right)\sqrt{1-\frac{v_{21}^{2}}{c^{2}}}\ , \tag{2.12}$$

where ψ – some function of a unique nontrivial invariant (2.9), therewith ψ is restricted at $r_{21} \to \infty$. Further for entry simplification we will omit a coefficient 21 and we will substitute $t_1 \equiv t$.

The minimality condition of action function *S* leads to the requirement of positivity of the second variation of action function:

$$\delta^2 S = \int dt \left\{ \tfrac{1}{2}(\delta r)^2 \frac{\partial^2 L}{\partial r^2} + \tfrac{1}{2}(\delta v)^2 \frac{\partial^2 L}{\partial v^2} + \delta r\, \delta v \frac{\partial^2 L}{\partial r\, \partial v} \right\}, \quad \delta v = -\frac{d}{dt}\delta r .$$

(2.13), (2.14)

Taking into account a vanishing of variations δv and δr on boundaries of an interval of integration, we obtain

$$\int dt\, \delta r\, \delta v \frac{\partial^2 L}{\partial r\, \partial v} = \tfrac{1}{2}\int dt\, (\delta r)^2 \left(-v \frac{\partial^3 L}{\partial v\, \partial r^2} + \dot{v} \frac{\partial^3 L}{\partial r\, \partial v^2} \right). \tag{2.15}$$

Presence of the term proportional $\dot{v}$, in expression for $\delta^2 S$ (2.13), (2.15) makes impossible minimization of function *S*. The absence of this term requires the following condition:

$$\frac{\partial^3 L}{\partial r\, \partial v^2} = 0. \tag{2.16}$$

Thus the requirement of a minimality of action function $\delta^2 S > 0$ (2.13), (2.15) leads to inequalities

$$\frac{\partial^2 L}{\partial v^2} > 0\,, \qquad \frac{\partial^2 L}{\partial r^2} - v \frac{\partial^3 L}{\partial v\, \partial r^2} > 0. \qquad (2.17), (2.18)$$

From a requirement (2.16) we get common expression for *L*:

$$L(v,r) = A(v) + B(r) + v\, D(r). \tag{2.19}$$

Comparing this expression with the formula (2.12) that at the account of equality (2.9) looks like:

$$L(v,r) = \psi\left(r \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \right) \sqrt{1-\frac{v^2}{c^2}}\ , \tag{2.20}$$

it is easy to see that in accordance to requirement (2.19) function ψ can be subscribed as

$$\psi(x) = E + R x + T \frac{1}{x}, \tag{2.21}$$

Here E, R, T – some constant values. In this expression it is necessary to set $R = 0$ because of a requirement of limitation of L at $r \to \infty$. Then it is possible to present expression $L(v,r)$ in a view

$$L(v,r) = -m_2 c^2 \left\{ \sqrt{1 - \frac{v^2}{c^2}} - \left(1 - \frac{v}{c}\right)\frac{r_0}{r} \right\}. \tag{2.22}$$

Here m_2 and r_0 are constants. As Lagrangian transformation

$$L \to L + \frac{d}{dt} f(r,t)$$

leaves invariable the first variation of S, at deriving of equations of motion it is possible to take advantage instead of the formula (2.22) of following expression

$$L_{21}(v_{21},r_{21}) = -m_2 c^2 \sqrt{1 - \frac{v_{21}^2}{c^2}} + m_2 c^2 \frac{r_0}{r_{21}}. \tag{2.23}$$

Let's note, however, that the given expression for L can be used only for the specified purpose.

It will be shown later that both the first (kinematical), and second (power) terms in a right part of expressions (2.22), (2.23) define action of a particle 2 on a particle 1.

Inequalities (2.17), (2.18) for a Lagrangian (2.22), (2.23) lead to the requirement of positivity of constants

$$m_2 > 0, \quad r_0 > 0. \tag{2.24}$$

Let's assume that for a considering case of a gravitational interaction the function of action of a particle 2 on a particle 1 depends only on mass of a particle 2, and that r_0 is the universal world length.

The constant of world length could define a space-time curvature. However, as it is easy to see, any dependence $c(r)$ leads to impossibility of sufficing of a minimality condition of action function (2.16). It means that introduction of curved space-time in the given theory is impossible.

Principle of minimum action and equations of motion

Equations of motion of system of particles are determined by a minimality condition of corresponding action function. *The special feature of the given theory is that it is impossible to define unified action function for all system of particles. Thus special action function should be defined for each distance.*

In this case we consider a motion of a particle 2 related to a particle 1 (i.e. distance $r_{21}(t)$) in the presence of other bodies k (Fig. 2.3). Action function which is necessary for varying on distance r_{21}, obviously involves all pair action functions of influence on a particle 2 at the moment $t_2 \equiv t_{21}$, and the same functions of influence on a particle 1 at the moment $t = t_1$:

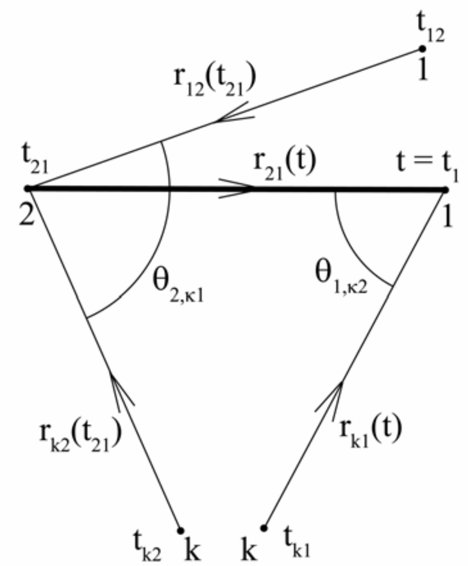

Fig. 2.3. Scheme of action on particles 1 and 2, defining the variation of distance r_{21}.

$$S[r_{21}] = S_{21} + S_{12} + \sum_{k \neq 1,2} (S_{k1} + S_{k2}), \quad S_{\alpha\beta} = \int L_{\alpha\beta}\, dt_\beta . \tag{2.25}$$

Here the Lagrangians are similar to $L_{\alpha\beta}$ (2.23). Corresponding moments of time are defined by causal relationships

$$t_{21} = t - \frac{r_{21}(t)}{c},\ t_{12} = t_{21} - \frac{r_{12}(t_{21})}{c},\ t_{k1} = t - \frac{r_{k1}(t)}{c},\ t_{k2} = t_{21} - \frac{r_{k2}(t_{21})}{c}. \tag{2.26}$$

The equations for r_{21} we derive, equating zero the first variation of $S[r_{21}]$ (2.25) on parameter r_{21}. Thus it is necessary to adjust all integrals on time (2.25) for uniform time t. Then the first variation of $S_{\alpha\beta}$ we will write in a view:

$$\delta S_{\alpha\beta} = -\int \delta r_{21} \cdot G_{\alpha\beta}\, dt, \quad G_{\alpha\beta} = m_\alpha \frac{\delta r_{\alpha\beta}}{\delta r_{21}} \left\{ \frac{d}{dt} \left[\dot{r}_{\alpha\beta} \left(\dot{t}_\beta^2 - \frac{\dot{r}_{\alpha\beta}^2}{c^2} \right)^{-1/2} \right] + \dot{t}_\beta \frac{c^2 r_0}{r_{\alpha\beta}^2} \right\}, \tag{2.27}$$

considering the expression

$$v_{\alpha\beta} = -\frac{d}{dt_\beta} r_{\alpha\beta},$$

that is similar to expression (2.22). And the point above parameter means a derivative on time t:

$$\dot{r}_{\alpha\beta} \equiv \frac{d}{dt} r_{\alpha\beta}, \quad \dot{t}_\beta \equiv \frac{d}{dt} t_\beta . \tag{2.28}$$

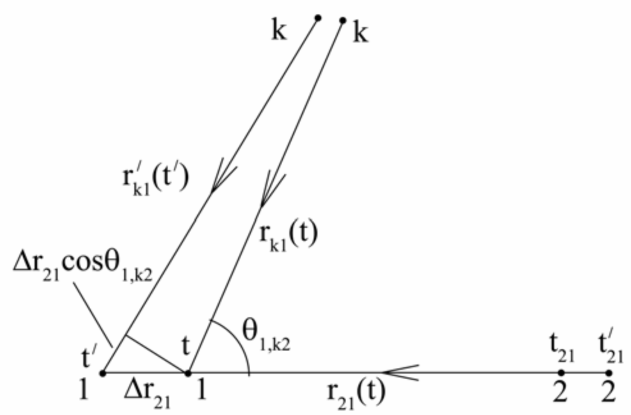

Fig. 2.4. Scheme of influence of "priming" shift Δr_{21} of a particle 1 on a modification of distances $r'_{21} - r_{21} = \delta r_{21}$ and $r'_{k1} - r_{k1} = \delta r_{k1}$.

Thus the vanishing of the first variation of $S[r_{21}]$ leads to the equation of motion for r_{21}:

$$G_{21} + G_{12} + \sum_{k \neq 1,2} (G_{k1} + G_{k2}) = 0. \qquad (2.29)$$

In this equation functions $G_{\alpha\beta}$ (2.27) are defined by variations of distances $\delta r_{\alpha\beta}$ on quantity δr_{21}.

Dependent variations of distances

Let's calculate at first a variation of r_{k1} on r_{21}. Let the particle 1 move along a direction $2 \to 1$ (Fig. 2.4), having transited distance Δr_{21} in time $t' - t$. As a consequence of motion of particle 2 during the period $t' - t$ we get the growth of r_{21} on corresponding distance Δr_{21}^{v}:

$$\Delta r_{21}^{v} \approx -v_{21}\left[(t' - t'_{21}) - (t - t_{21})\right] = -\delta r_{21} \frac{v_{21}(t)}{c}. \qquad (2.30)$$

Thus, the complete change of r_{21} is equal to

$$\delta r_{21} = \Delta r_{21} + \Delta r_{21}^{v} = \Delta r_{21} - \delta r_{21} \frac{v_{21}(t)}{c}, \qquad (2.31)$$

Therefore we have:

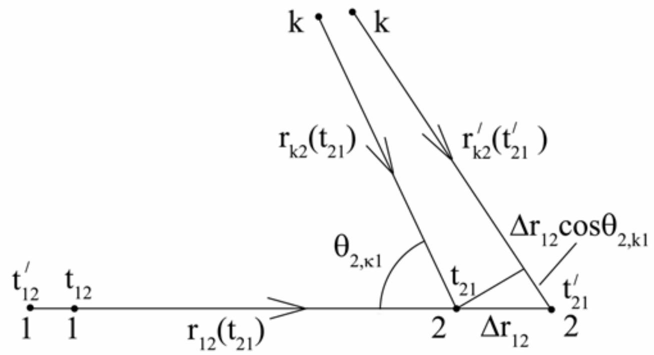

Fig. 2.5. Scheme of influence of "priming" shift Δr_{12} of a particle 2 on a modification of distances $r'_{12} - r_{12} = \delta r_{12}$ and $r'_{k2} - r_{k2} = \delta r_{k2}$.

$$\delta r_{21} = \frac{\Delta r_{21}}{1+\frac{v_{21}(t)}{c}}. \tag{2.32}$$

Moving of a particle 1 along a direction $2 \to 1$ leads to shift of distance r_{k1} equal to $\Delta r_{21}\cos\theta_{1,k2}$, so we obtain a complete modification δr_{k1} taking into account a motion of particle k with the velocity $v_{k1}(t)$, similar to (2.31),

$$\delta r_{k1} \approx \Delta r_{21}\cos\theta_{1,k2} - \delta r_{k1}\frac{v_{k1}(t)}{c},$$

whence we have:

$$\delta r_{k1} \approx \frac{\Delta r_{21}\cos\theta_{1,k2}}{1+\frac{v_{k1}(t)}{c}}. \tag{2.33}$$

Comparing expressions (2.32) and (2.33), we gain

$$\frac{\delta r_{k1}}{\delta r_{21}} = \frac{1+\frac{v_{21}(t)}{c}}{1+\frac{v_{k1}(t)}{c}}\cos\theta_{1,k2}. \tag{2.34}$$

Similarly we get a variation of r_{k2} on r_{12} (not r_{21}!) (Fig. 2.5):

$$\frac{\delta r_{k2}}{\delta r_{12}} = \frac{1+\frac{v_{12}(t_{21})}{c}}{1+\frac{v_{k2}(t_{21})}{c}}\cos\theta_{2,k1}. \tag{2.35}$$

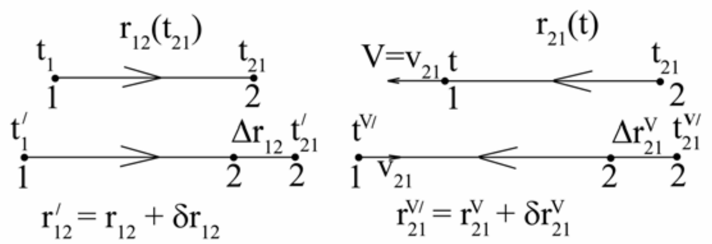

Fig. 2.6. Scheme of influence of "priming" shift Δr_{12} of a particle 2 in the reference system of this particle, on the variations of distances r_{12} and r_{21}.

Let's calculate a variation r_{k2} on r_{21}. For this purpose it is necessary to calculate a variation r_{12} on r_{21}, further, to present a variation r_{k2} on r_{21} in a view:

$$\frac{\delta r_{k2}}{\delta r_{21}} = \frac{\delta r_{12}}{\delta r_{21}} \cdot \frac{\delta r_{k2}}{\delta r_{12}}. \tag{2.36}$$

Let's derive a variation $\delta r_{12} / \delta r_{21}$. Let the particle 2 move along a direction $1 \to 2$ having a transit distance equal Δr_{12} (Fig. 2.6). Then the respective alteration of distance r_{12} (equal δr_{12}, similar to (2.32), looks like:

$$\delta r_{12} \approx \Delta r_{12} - v_{12}(t_{21}) \cdot \Delta t_{12}\,, \quad \Delta t_{12} = (t'_2 - t'_1) - (t_2 - t_1) = \frac{\delta r_{12}}{c},$$

whence we gain

$$\delta r_{12} \approx \frac{\Delta r_{12}}{1 + \frac{v_{12}(t_{21})}{c}}. \tag{2.37}$$

By deriving variation δr_{21} let's go over in a reference system of the particle 2. Such system moves with velocity $V = v_{21}$ related to a particle 1 (Fig. 2.6). We will designate all parameters in such reference system an upper index *V*. As the particle 2 does not move in this system, parameter Δr^V_{21} is equal to Δr_{12}. Thus the distance variation δr^V_{21} is related to the formula Δr^V_{21}, similar to (2.37):

$$\delta r^V_{21} = \frac{\Delta r^V_{21}}{1 + \frac{v_{21}(t)}{c}} = \frac{\Delta r_{12}}{1 + \frac{v_{21}(t)}{c}}. \tag{2.38}$$

Quantity δr^V_{21} is related to δr_{21} by transformation rule of distances (2.11) at $V = v_{21}$:

$$\delta r_{21}^{V} = \delta r_{21}\sqrt{\left(1+\frac{v_{21}(t)}{c}\right)\Big/\left(1-\frac{v_{21}(t)}{c}\right)}\ .$$

Thus, from this relation and expressions (2.37), (2.38) we get:

$$\frac{\delta r_{12}}{\delta r_{21}} = \frac{1+\dfrac{v_{21}(t)}{c}}{1+\dfrac{v_{12}(t_{21})}{c}}\sqrt{\frac{1+\dfrac{v_{21}(t)}{c}}{1-\dfrac{v_{21}(t)}{c}}}\,,$$

$$\frac{\delta r_{k2}}{\delta r_{21}} = \frac{1+\dfrac{v_{21}(t)}{c}}{1+\dfrac{v_{k2}(t_{21})}{c}}\sqrt{\frac{1+\dfrac{v_{21}(t)}{c}}{1-\dfrac{v_{21}(t)}{c}}}\cos\theta_{2,k1}\,. \tag{2.39}$$

As it is clear from formulas (2.25), (2.26), we must use following expressions for quantities $\dot{t}_{\beta}$:

$$\frac{dt_1}{dt} \equiv 1, \qquad \frac{dt_2}{dt} \equiv \frac{dt_{21}}{dt} = 1-\frac{\dot{r}_{21}}{c}, \tag{2.40}$$

as like as corresponding expressions for velocities:

$$v_{21} = \dot{r}_{21}\,, v_{k1} = -\dot{r}_{k1}\,, v_{12} = -\frac{dr_{12}}{dt_{21}} = -\frac{\dot{r}_{12}}{1-\dfrac{\dot{r}_{21}}{c}}\,, v_{k2} = -\frac{dr_{k2}}{dt_{21}} = -\frac{\dot{r}_{k2}}{1-\dfrac{\dot{r}_{21}}{c}}\,.$$

Thus dependent variations of distances take a form:

$$\frac{\delta r_{k1}}{\delta r_{21}} = \frac{1-\dfrac{\dot{r}_{21}}{c}}{1-\dfrac{\dot{r}_{k1}}{c}}\cos\theta_{1,k2}\,, \quad \frac{\delta r_{12}}{\delta r_{21}} = \frac{\left(1-\dfrac{\dot{r}_{21}}{c}\right)^2}{1-\dfrac{\dot{r}_{21}+\dot{r}_{12}}{c}}\sqrt{\frac{1-\dfrac{\dot{r}_{21}}{c}}{1+\dfrac{\dot{r}_{21}}{c}}}\,,$$

$$\frac{\delta r_{k2}}{\delta r_{21}} = \frac{\left(1-\dfrac{\dot{r}_{21}}{c}\right)^2}{1-\dfrac{\dot{r}_{21}+\dot{r}_{k2}}{c}}\sqrt{\frac{1-\dfrac{\dot{r}_{21}}{c}}{1+\dfrac{\dot{r}_{21}}{c}}}\cos\theta_{2,k1}\,. \tag{2.41}$$

Substituting expressions (2.40), (2.41) in formulas for $G_{\alpha\beta}$ (2.27), we obtain:

$$G_{21} = m_2\left\{\ddot{r}_{21}\left(1-\frac{\dot{r}_{21}^2}{c^2}\right)^{-3/2} + \frac{c^2\, r_0}{r_{21}^2}\right\}, \tag{2.42}$$

$$G_{12}=m_1\frac{\left(1-\frac{\dot r_{21}}{c}\right)^3}{1-\frac{\dot r_{21}+\dot r_{12}}{c}}\sqrt{\frac{1-\frac{\dot r_{21}}{c}}{1+\frac{\dot r_{21}}{c}}}\times$$
$$\times\left\{\left[\ddot r_{12}\left(1-\frac{\dot r_{21}}{c}\right)+\ddot r_{21}\frac{\dot r_{12}\,\dot r_{21}}{c^2}\right]\left[\left(1-\frac{\dot r_{21}}{c}\right)^2-\frac{\dot r_{12}^2}{c^2}\right]^{-3/2}+\frac{c^2\,r_0}{r_{12}^2}\right\}, \tag{2.43}$$

$$G_{k1}=m_k\cos\theta_{1,k2}\frac{1-\frac{\dot r_{21}}{c}}{1-\frac{\dot r_{k1}}{c}}\left\{\ddot r_{k1}\left(1-\frac{\dot r_{k1}^2}{c^2}\right)^{-3/2}+\frac{c^2\,r_0}{r_{k1}^2}\right\}, \tag{2.44}$$

$$G_{k2}=m_k\cos\theta_{2,k1}\frac{\left(1-\frac{\dot r_{21}}{c}\right)^3}{1-\frac{\dot r_{k2}+\dot r_{21}}{c}}\sqrt{\frac{1-\frac{\dot r_{21}}{c}}{1+\frac{\dot r_{21}}{c}}}\times$$
$$\times\left\{\left[\ddot r_{k2}\left(1-\frac{\dot r_{21}}{c}\right)+\ddot r_{21}\frac{\dot r_{k2}\,\dot r_{21}}{c^2}\right]\left[\left(1-\frac{\dot r_{21}}{c}\right)^2-\frac{\dot r_{k2}^2}{c^2}\right]^{-3/2}+\frac{c^2\,r_0}{r_{k2}^2}\right\}. \tag{2.45}$$

Thus, the equations for r_{21} (2.29), (2.42) – (2.45) contain derivatives not only from r_{21}, but also from all remaining distances which describe action on a particle 1 (in the moment *t*) and on a particle 2 (in the moment t_{21}). In such a way it is possible to derive the equations for distances r_{12}, r_{k1}, r_{k2} etc.

3. Slow motion of pair of particles interacting with distant, massive, almost motionless bodies

Let's view a motion of pair of particles 1 and 2 with masses m_1 and m_2, the distance between which is much less than distances to massive bodies with numbers *k* $(m_k >> m_1,\, m_2)$

$$r_{21} << r_{k1},\, r_{k2}\,. \tag{3.1}$$

The requirement of a slow motion of particles $\dot r_{21} << c$ leads to the characteristic time $\tau_{21} \sim r_{21}/\dot r_{21}$ of a modification of distance r_{21} considerably exceeds time for which light propagates this distance

$$\tau_{21} >> r_{21}/c. \tag{3.2}$$

Let's assume further that the characteristic time of a changing of distance between distant bodies considerably exceeds τ_{21}.

The equation for r_{21}

In a limit of small relative velocities of all particles $\left(\dot{r}_{\alpha\beta} << c\right)$ it is possible to set $r_{12} \approx r_{21}$, and from expressions (2.29), (2.41) – (2.45) it is gained a following equation for r_{21}:

$$\left(m_1 + m_2\right)\ddot{r}_{21} + \sum_k m_k \left(\ddot{r}_{k1}\cos\theta_{1,k2} + \ddot{r}_{k2}\cos\theta_{2,k1}\right) =$$

$$= \left(m_1 + m_2\right)F\left(r_{12}\right) + \sum_k m_k \left[F\left(r_{k1}\right)\cos\theta_{1,k2} + F\left(r_{k2}\right)\cos\theta_{2,k1}\right]. \tag{3.3}$$

Here the force function $F(r)$ looks like

$$F(r) = -\frac{c^2 r_0}{r^2}. \tag{3.4}$$

The particular interest represents a second term in the left part of equation (3.3). This term is related to action of far *k*-bodies on a pair of particles 1 and 2, and this action carries not "force", but the kinematical character which has been caused by correlation between distances r_{21} and r_{k1}, r_{k2}. As shown later, this action of remote *k*-bodies leads to centrifugal force origin at mutual rotation of pair of particles concerning remote bodies.

***The equations for* and** r_{k1} **and** r_{k2}

The equations for r_{k1} and r_{k2} look similar to (2.29), (2.27):

$$G_{k1} + G_{21} + \sum_{n \neq k} G_{n1} + G_{1k}^{(1)} + G_{2k}^{(1)} + \sum_{n \neq k} G_{nk}^{(1)} = 0, \tag{3.5}$$

$$G_{k2} + G_{12} + \sum_{n \neq k} G_{n2} + G_{2k}^{(2)} + G_{1k}^{(2)} + \sum_{n \neq k} G_{nk}^{(2)} = 0. \tag{3.6}$$

Schemes of actions are given, accordingly, on Fig. 3.1, 3.2. From these schemes it is possible to see that terms $G_{1k}^{(1)}, G_{2k}^{(1)}$ in the left part of equation (3.5) and terms $G_{2k}^{(2)}, G_{1k}^{(2)}$ – in equation (3.6) are related to positions of particles 1 and 2 in the

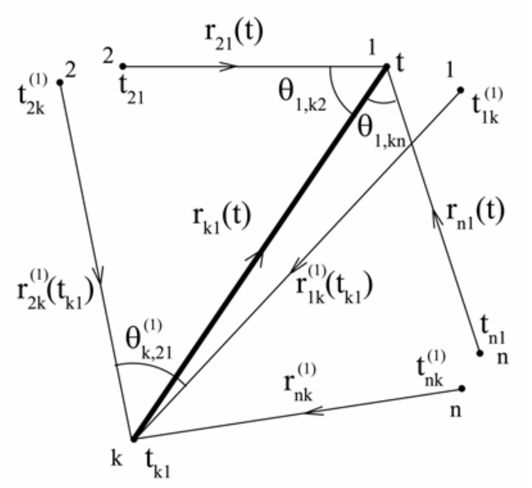

Fig. 3.1. Scheme of influence on particles k and 1, determining the modification of distance r_{k1} .

moments, accordingly, $t_{1k}^{(1)}, t_{2k}^{(1)}$ and $t_{1k}^{(2)}, t_{2k}^{(2)}$. This times exceed to quantities $2r_{k1}/c$, $2r_{k2}/c$ the moment t. As we consider rather remote k-bodies (for example galaxies distances to which considerably exceed millions light years), it is clear that phases of motion of particles 1 and 2 in such early moments of time in any way are not correlated with a phase in the moment t, and besides they will be essentially different for different k-bodies. Therefore contributions G_{1k} and G_{2k} in the equations (3.5), (3.6) will lead to almost accidental influence on motion of particles 1 and 2 in the moment t. Further we will neglect such quasiaccidental influence, setting

$$G_{1k}^{(1)} = G_{2k}^{(1)} = G_{1k}^{(2)} = G_{2k}^{(2)} = 0 .$$

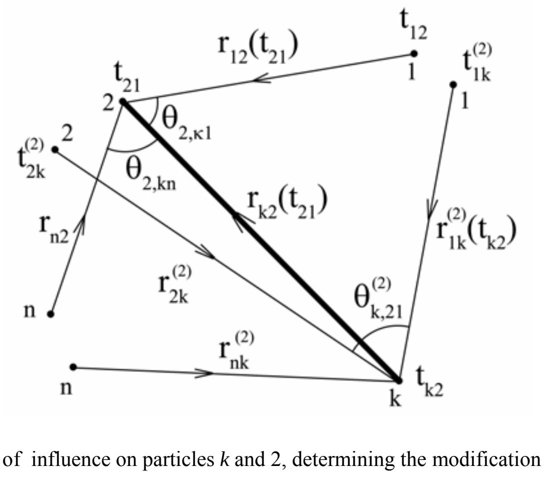

Fig. 3.2. Scheme of influence on particles k and 2, determining the modification of distance r_{k2}.

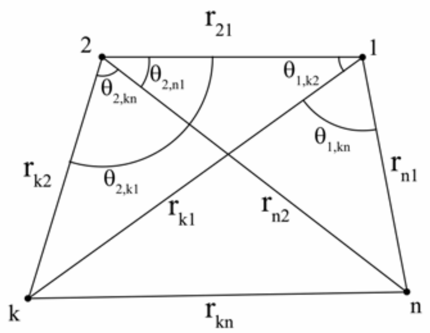

Fig. 3.3. Scheme of influence on particles 1 and 2 in case of slow motion when relationship of angles and distances are defined by an Euclidean geometry.

Let's assume also that the characteristic times of the relative motion of remote *k*-bodies considerably exceed time τ_{21} for particles 1 and 2. Accordingly we shall completely neglect the interaction between remote bodies, putting $G_{nk}^{(1)}=0,\ G_{nk}^{(2)}=0$ in the equations (3.5), (3.6).

Let's consider, as well as above for the equation (3.3), a case of small relative velocities, when $\dot{r}_{\alpha\beta} << c$. In this limit a relation between angles and distances is defined by an Euclidean geometry. Thus action schemes on Fig. (3.1), (3.2) are exchanged by the scheme on Fig. 3.3, and the equations (3.5), (3.6) become:

$$m_k\,\ddot{r}_{k1}+m_2\,\ddot{r}_{21}\cos\theta_{1,2k}+\sum_{n\neq k} m_n\,\ddot{r}_{n1}\cos\theta_{1,kn}=$$

$$=m_k\,F(r_{k1})+m_2\,F(r_{21})\cos\theta_{1,2k}+\sum_{n\neq k} m_n\,F(r_{n1})\cos\theta_{1,kn}\,, \qquad (3.7)$$

$$m_k\,\ddot{r}_{k2}+m_1\,\ddot{r}_{21}\cos\theta_{2,k1}+\sum_{n\neq k} m_n\,\ddot{r}_{n2}\cos\theta_{2,k1}=$$

$$=m_k\,F(r_{k2})+m_1\,F(r_{21})\cos\theta_{2,k1}+\sum_{n\neq k} m_n\,F(r_{n2})\cos\theta_{1,kn}\,. \qquad (3.8)$$

Further, it is advisable to use the transition to new variables (Fig. 3.4). We will take advantage of following decompositions of parameters at small rations $r_{21}/r_k << 1$:

$$r_{21}=r_1+r_2,\quad r_{k1}\approx r_k+r_1\cos\varphi_k,\quad r_{k2}\approx r_k-r_2\cos\varphi_k,\quad r_{n1}\approx r_n+r_1\cos\varphi_n,$$

$$\cos\theta_{2,k1}\approx-\cos\varphi_k+\frac{r_2}{r_k}\sin^2\varphi_k\,,\quad \cos\theta_{1,k2}\approx\cos\varphi_k+\frac{r_1}{r_k}\sin^2\varphi_k\,,$$

$$\cos\theta_{1,kn}\approx\cos\varphi_{kn}\left(1-\frac{r_1}{r_k}\cos\varphi_k-\frac{r_1}{r_n}\cos\varphi_n\right)+\frac{r_1}{r_n}\cos\varphi_k+\frac{r_1}{r_k}\cos\varphi_n\,,$$

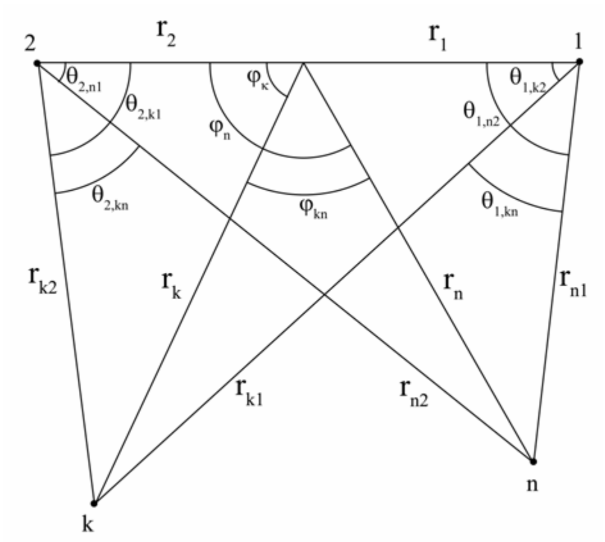

Fig. 3.4. To the replacement of distances and angles in the equations for r_{21} , r_{k1}, r_{k2} The planes formed by triads of particles, generally speaking, are not coplanar.

$$\cos\theta_{2,kn} \approx \cos\varphi_{kn}\left(1+\frac{r_2}{r_k}\cos\varphi_k+\frac{r_2}{r_n}\cos\varphi_n\right)-\frac{r_2}{r_n}\cos\varphi_k-\frac{r_2}{r_k}\cos\varphi_n\ . \qquad (3.9)$$

Substituting quantities (3.9) in the equation (3.3) and neglecting the terms, quadratic in r_{21}, we obtain the equation for r_{21}:

$$(m_1+m_2)\ddot{r}_{21}+\sum_k m_k\left[r_{21}\frac{\ddot{r}_k}{r_k}\sin^2\varphi_k+\cos\varphi_k\,\ddot{\overline{r_{21}\cos\varphi_k}}\right]=$$

$$=(m_1+m_2)F(r_{21})+r_{21}\sum_k m_k\frac{1}{r_k}F(r_k)\left(1-3\cos^2\varphi_k\right). \qquad (3.10)$$

In the same limit the equation for r_{k1} becomes:

$$m_k\ddot{r}_k+m_k\,\ddot{\overline{r_1\cos\varphi_k}}+m_2\ddot{r}_{21}\cos\varphi_k+\sum_{n\neq k}m_n\cos\varphi_{kn}\,\ddot{\overline{r_1\cos\varphi_n}}+$$

$$+\sum_{n\neq k}m_n\left\{\ddot{r}_n\left[\cos\varphi_{kn}\left(1-\frac{r_1}{r_k}\cos\varphi_k-\frac{r_1}{r_n}\cos\varphi_n\right)+\frac{r_1}{r_k}\cos\varphi_n+\frac{r_1}{r_n}\cos\varphi_k\right]\right\}=$$

$$= m_k F(r_k) - 2 m_k F(r_k) \frac{r_1}{r_k} \cos\varphi_k + m_2 F(r_{21}) \left[\cos\varphi_k + \frac{r_1}{r_k} \sin^2 \varphi_k \right] +$$

$$+ \sum_{n \neq k} m_n F(r_n) \left\{ \cos\varphi_{kn} \left(1 - \frac{r_1}{r_k} \cos\varphi_k - \frac{r_1}{r_n} \cos\varphi_n \right) \right\} +$$

$$+ \sum_{n \neq k} m_n F(r_n) \left\{ \frac{r_1}{r_n} cos\,\varphi_k + \frac{r_1}{r_k} cos\,\varphi_n - 2 \frac{r_1}{r_n} cos\,\varphi_{kn}\, cos\,\varphi_n \right\}. \quad (3.11)$$

Accordingly we get the equation for r_{k2}:

$$m_k \ddot{r}_k - m_k \overline{\ddot{r}_2\, cos\,\varphi_k} - m_1 \ddot{r}_{21}\, cos\,\varphi_k - \sum_{n \neq k} m_n\, cos\,\varphi_{kn} \overline{\ddot{r}_2\, cos\,\varphi_n} +$$

$$+ \sum_{n \neq k} m_n \left\{ \ddot{r}_n \left[\cos\varphi_{kn} \left(1 + \frac{r_2}{r_k} \cos\varphi_k + \frac{r_2}{r_n} \cos\varphi_n \right) - \frac{r_2}{r_n} \cos\varphi_k - \frac{r_2}{r_k} \cos\varphi_n \right] \right\} =$$

$$= m_k F(r_k) + 2 m_k F(r_k) \frac{r_2}{r_k} \cos\varphi_k + m_1 F(r_{21}) \left[-\cos\varphi_k + \frac{r_2}{r_k} \sin^2 \varphi_k \right] +$$

$$+ \sum_{n \neq k} m_n F(r_n) \left\{ \cos\varphi_{kn} \left(1 + \frac{r_2}{r_k} \cos\varphi_k + \frac{r_2}{r_n} \cos\varphi_n \right) \right\} +$$

$$+ \sum_{n \neq k} m_n F(r_n) \left\{ -\frac{r_2}{r_k} cos\,\varphi_n - \frac{r_2}{r_n} \cos\varphi_k + 2 \frac{r_2}{r_n} \cos\varphi_n \cos\varphi_{kn} \right\}. \quad (3.12)$$

Subtracting accordingly left and right parts of the equations (3.11), (3.12), we obtain:

$$(m_1 + m_2) \ddot{r}_{21} \cos\varphi_k + \sum_n m_n \overline{\ddot{r}_{21} \cos\varphi_n} \cos\varphi_{kn} +$$

$$+ r_{21} \sum_n m_n \ddot{r}_n \left[-\cos\varphi_{kn} \left(\frac{\cos\varphi_k}{r_k} + \frac{\cos\varphi_n}{r_n} \right) + \frac{\cos\varphi_n}{r_k} + \frac{\cos\varphi_k}{r_n} \right] =$$

$$= (m_1 + m_2) F(r_{21}) \cos\varphi_k +$$

$$+ r_{21} \sum_n m_n F(r_n) \left[-\cos\varphi_{kn} \left(\frac{\cos\varphi_k}{r_k} + \frac{\cos\varphi_n}{r_n} \right) \right] +$$

$$+ r_{21} \sum_n m_n F(r_n) \left[\frac{\cos\varphi_k}{r_n} + \frac{\cos\varphi_n}{r_k} - 2 \frac{\cos\varphi_n \cos\varphi_{kn}}{r_n} \right]. \quad (3.13)$$

Adding accordingly left and right parts of the equations (3.11), (3.12), we get a following equation $(\cos\varphi_{kk} \equiv 1)$:

$$2\sum_n m_n \ddot{r}_n \cos\varphi_{kn} - 2\sum_n m_n F(r_n)\cos\varphi_{kn} =$$

$$= -\ddot{r}_{21}(m_2 - m_1)\cos\varphi_k + (m_2 - m_1)F(r_{21})\cos\varphi_k -$$

$$-\sum_n m_n \overline{(r_1 - r_2)\cos\varphi_k}^{\,\cdot\cdot} \cos\varphi_{kn} -$$

$$-(r_1 - r_2)\sum_n m_n \ddot{r}_n \left[-\cos\varphi_{kn}\left(\frac{\cos\varphi_k}{r_k} + \frac{\cos\varphi_n}{r_n}\right) + \frac{\cos\varphi_k}{r_n} + \frac{\cos\varphi_n}{r_k}\right] -$$

$$-(r_1 - r_2)\sum_n m_n F(r_n)\left[\cos\varphi_{kn}\left(\frac{\cos\varphi_k}{r_k} + \frac{\cos\varphi_n}{r_n}\right)\right] -$$

$$-(r_1 - r_2)\sum_n m_n F(r_n)\left[-\frac{\cos\varphi_k}{r_n} - \frac{\cos\varphi_n}{r_k} + 2\frac{\cos\varphi_n \cos\varphi_{kn}}{r_n}\right], \tag{3.14}$$

where summation on n includes $n = k$.

It is easy to see that choosing the ration of lengths of segments r_1 and r_2 as following

$$m_1 r_1 = m_2 r_2 , \tag{3.15}$$

corresponding to a centre of masses in the Newtonian mechanics, the right part of the equation (3.14) is converted in zero in accordance to the equation (3.13). Thus the equation (3.14) takes a form:

$$\sum_n m_n \ddot{r}_n \cos\varphi_{kn} = \sum_n m_n F(r_n)\cos\varphi_{kn} . \tag{3.16}$$

Equations of motion of pair of particles 1 and 2 at the isotropic space allocation of massive distant bodies

Let's consider the unit vector **e** in the three-dimensional space, directed from a particle 1 to particle 2, and vector $\mathbf{e}_k$ directed from a centre of mass to a body k. Then the sum in the right part of the equation (3.16) we will write in a view:

$$\sum_n m_n F(r_n)\cos\varphi_{kn} = \mathbf{e}_k \cdot \sum_n m_n F(r_n)\mathbf{e}_n .$$

Let's consider that the isotropy requirement is satisfied

$$\sum_n m_n F(r_n)\mathbf{e}_n = 0 . \tag{3.17}$$

Then the equation (3.16) takes a form:

$$\mathbf{e}_k \cdot \sum_n m_n \ddot{r}_n \mathbf{e}_n = 0 . \tag{3.18}$$

Let's view shift of a centre of masses

$$\mathbf{r}_n(t) = \mathbf{r}_n^0 + \mathbf{R}(t) ,$$

where $R << r_n$. Meaning a small modifications of vectors $\mathbf{e}_n$ (of order r/r_n), we get for r_n:

$$r_n(t) \approx r_n^0 + \mathbf{e}_n \mathbf{R}(t) .$$

Substituting this expression in the equation (3.18), we have:

$$\sum_n m_n \left(\ddot{\mathbf{R}} \mathbf{e}_n\right) \mathbf{e}_n = 0 . \tag{3.19}$$

Let's consider satisfied a following requirement of an isotropy of a masses distribution of distant bodies

$$\sum_n m_n e_{n,i} e_{n,j} = \delta_{ij} \tfrac{1}{3} \sum_n m_n . \tag{3.20}$$

Thus from a relation (3.19) we get an equation of motion of a centre of masses of particles 1 and 2

$$\ddot{\mathbf{R}} = 0, \quad \ddot{\mathbf{r}}_n = 0 . \tag{3.21}$$

Let's view further a difference equation (3.13). We use the formula (3.21) for $\ddot{r}_n$, requirements of an isotropy (3.17), (3.19), and also one more requirement of an isotropy

$$\sum_n m_n \frac{1}{r_n} F(r_n) e_{n,i} e_{n,j} = \delta_{ij} \tfrac{1}{3} \sum_n m_n \frac{1}{r_n} F(r_n) . \tag{3.22}$$

Then neglecting by the first term in comparison with second term in the left part of equation (3.13) $\left(m_1, m_2 << m_k\right)$, we obtain from the equation (3.13):

$$\ddot{\mathbf{r}}_{21} = \mathbf{e}(m_1 + m_2) f(r_{21}) , \tag{3.23}$$

$$\mathbf{e} = \frac{\mathbf{r}_{21}}{r_{21}} , \quad f(r_{21}) = -\frac{\gamma}{r_{21}^2} , \quad \gamma = \frac{3c^2 r_0}{\sum_n m_n} . \tag{3.24}$$

Let's note that at performance of requirements of an isotropy equations (3.20), (3.22) for r_{21} (3.10) coincides with the equation (3.23) scalar multiplied on the right and the left parts by a vector **e**. It is easy to see that relations (3.21), (3.23) coincide with the Newtonian equations of motion if to consider γ (3.24) as gravitation constant of a Newtonian mechanics. Thus from the equation (3.23) we have the usual equation for the relative distance *r* (in cylindrical coordinates r, φ):

$$\ddot{r} - r(\dot{\varphi})^2 = (m_1 + m_2) f(r) \tag{3.25}$$

and an orbital momentum conservation law $r^2\dot{\varphi} = \mathrm{const}$. We will underline that both the first term of the left part of equation (3.25), and second (centrifugal force) are caused by action of almost immobile, massive, distant bodies on a pair of particles 1 and 2. Thus a gravitation constant γ (3.24) is inversely proportional to total of masses of such distant bodies.

Let's underline once again that at a deduction of motion equations of pair of particles (3.21), (3.23) the performance of three requirements of an isotropy of space allocation of distant bodies (3.17), (3.20), (3.22) was supposed.

Let's point out also the interesting fact. Apparently from the equation (3.25), both in the Newtonian, and in the viewed theory the relative motion is defined by the *sum* of masses of particles $m_1 + m_2$. It means, what exactly in the Newtonian equation for the relative distance *r* remains "memory" about that actually the attraction of bodies is the sum of mutual actions of particles, each is proportional to a mass of an acting particle.

4. Deductions

1. For the first time the gravitation theory is constructed on a relativity principle as it was understood by Berkeley, Mach and Poincaré, i.e. without using concept of absolute space (aether), contrary to the theory of universal gravitation of Newton and the known theories of gravitation, including a common relativity theory of Hilbert–Einstein.

2. On the basis of principles of a relativity, causality and limitation of velocities by quantity *c* the attraction law was obtained between particles at which forces of mutual gravitational action are not equal and proportional to masses of an acting particle. This law essentially differs from the Newtonian where the attractive force is proportional to product of masses of particles.

3. The causality principle leads to nonreversible in time equations of motion of particles. For the viewed problem of motion of pair of particles related to "far stars" the reversible equations in time were gained. That, however, is related to the assuming of small velocities of particles in comparison with a velocity of light and small characteristic times of particles motion in comparison with time of a modification of a configuration of "far stars".

4. It is discovered new – the kinematical – kind of action of the bodies, caused not by gravitational force, but by the relative acceleration of particles (and bodies) in conditions where motions depend only on relative distances and velocities. The centrifugal force is conditioned by this type of action for mutual rotation of particles related to far massive bodies.

5. Centrifugal force presence was calculated in respect to an isotropy of space allocation of far bodies, and implementation of three various conditions of an isotropy is required. In this case the equations of the Newtonian type for motion of pair of particles are obtained.

6. At essential removal from centre of an isotropy the centrifugal force can appear much less then the one calculated on the Newtonian theory. It can explain a possibility of rotation of objects with an angular velocity, acceding the values that Newton's theory supposes, without referring to ideas about “the latent masses”.

7. The centrifugal force modification on large distances from “Universe centre” can lead to essential modifications of a structure and spectrums of atoms and molecules.

8. At the account of anisotropy of allocation of distant bodies in a kinematical and force terms of equation for the relative distance there occurs a force proportional to anisotropy tensor .

9. The gravitation constant in "the Newtonian" limit appears to be inversely proportional to the sum of masses of distant bodies of the Universe. Finiteness of γ specifies in limitation of the total mass of such bodies.

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References

1. I. Newton. Philosophiae Naturalis Principia Mathematica. 1st Ed., London, 1686.
2. G.Berkeley De Motu Sive de Motus Principio et Natura et de Causa Communicationis Motuum. 1721.
3. E. Mach Die Mechanik in Ihrer Entwicklung. I Aufl. Leipzig, 1883.
4. H. Poincaré La science et l'hypothèse. Paris: Flammarion, 1902
5. Gravitation and Relativity. Ed. H.-Y. Chiu, W.F. Hoffmann. W.A. Benjamin, N.Y.-Amsterdam, 1964.
6. H. Poincaré Sur la dynamique de l'èlectron. Rendiconti del Circolo Matematico di Palermo. 1906. XXI. P.129-176.